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III. STRUCTURE OF CLASSICAL AND QUANTIZED FIELDS AND RELATED MATHEMATICAL TECHNIQUES

1. Quantum Field Theory

H. REEH — Additive Conservation Laws for Fields with Spin 143

W. LÜCKE — On Possible Relaxations of Strict Locality in Relativistic Quantum Field Theory 163

K. V. RERIKH — Chew-Low Equations as the Cremona Transformation. Structure of the First Integrals and General Solution 170

Y. OHNUKI, S. KAMEFUCHI — Klein Transformations for Generalized Numbers 179

2. Gauge Theory

A. ODZIJEWICZ — About a New Approach to the Yang-Mills Equations 191

E. R. NISSIMOV, S. J. PACHEVA — Axial Anomalies in Odd Dimensions 198

J. MICKELSSON — Electroweak Interactions from Space-Time Geometry 210

A. ASADA — Flat Connections of Differential Operators and Related Characteristic Classes 220

3. General Relativity

J. F. POMMARET — Gauge Theory and General Relativity 235

R. PERCACCI — Role of Soldering in Gravity Theory 250

M. GÜRSES — Integrability of the Vacuum Einstein Field Equations 260

E. BINZ — On the Structure of Euclidean Isometric Immersions 278

4. Infinite-Dimensional Lie-Algebras

I. T. TODOROV — Infinite Lie Algebras in 2-Dimensional Conformal Field Theory 297

V. K. DOBREV — Multiplets of Indecomposable Highest Weight Modules Over Infinite-Dimensional Lie Algebras: The Virasoro - $A_1^{(1)}$ Correspondence 348

AXIAL ANOMALIES IN ODD DIMENSIONS

E.R. Nissimov and S.J. Pacheva
Institute of Nuclear Research and Nuclear Energy
Boul. Lenin 72, 1784 Sofia, Bulgaria

1. One of the central concepts of modern quantum field theory - symmetry breaking, is generally considered in two principal settings:

(a) Spontaneous breaking (preferably dynamical, i.e. as a result of (nonperturbative) quantum effects) - when, due to a degeneracy of the ground state, the correlation functions do not respect the symmetry of the classical Lagrangian for some range of the coupling parameters. At certain critical values of the latter through phase transitions the original symmetry may be restored or further changed.

(b) Anomalous breaking - unavoidable violation of the classical symmetry due to regularization of the ultraviolet divergences on quantum level. This violation does not depend on the values of the coupling parameters and it cannot be repaired by adding finite local (in the fields) counterterms to the classical Lagrangian.

Most familiar are the chiral anomalies of gauge theories with left-(right-)handed fermions [1] in even space-time dimensions D (we shall consider Euclidean space-time), which may be summarized by the following equation for the functional determinant of the chiral Dirac operator $\not{\nabla}_{L,R}(A)$ in an external U(n)-gauge field $A_\mu(x)$ [2,3] (the cases O(n), Sp(n) may be treated analogously):

$$\ln\det[-i\not{\nabla}_{L,R}(A^g)] - \ln\det[-i\not{\nabla}_{L,R}(A)] = \mp 2\pi i \int_{\mathbb{R}_+^{D+1}} d^{D+1}X \left[\omega^{(D+1)}_{ChS}(A^g) - \omega^{(D+1)}_{ChS}(A)\right] . \qquad (1)$$

In (1) the following notations are used:

$$\not{\nabla}_{L,R}(A) = \gamma_\mu(\partial_\mu + i\Pi_{L,R}A_\mu(x)) \quad , \quad \Pi_{L,R} = \frac{1}{2}\left[1\!\!1 \pm \gamma^{(D+1)}\right] \quad ;$$

$$\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu} \quad , \quad \gamma^*_\mu = -\gamma_\mu \quad , \quad \gamma^{(D+1)} = i^{\frac{1}{2}D(D+1)}\gamma_0 \ldots \gamma_{D-1} = \left[\gamma^{(D+1)}\right]^* ,$$

$$\gamma^{(D+1)} = \pm 1\!\!1 \qquad (\text{for } D = \text{odd}) \quad ;$$

$$x \equiv (x^0,\ldots,x^{D-1}) \in \mathbb{R}^D \quad , \quad X \equiv (x,x^D) \in \mathbb{R}^{D+1} \quad , \quad \mathbb{R}_+^{D+1} = \{X; x^D \geqq 0\} \quad ;$$

$$A_\mu(x) = -i\,h^{-1}(x_\infty)(\partial_\mu h)(x_\infty) + O(|x|^{-1-\varepsilon}) \quad \text{for } |x| \longrightarrow \infty \quad , \quad x_\infty \in S_\infty^{D-1} ,$$

$$h: S_\infty^{D-1} \longrightarrow U(n) \quad , \quad \text{i.e. } F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] = O(|x|^{-2-\varepsilon}) \quad ;$$

$$(2)$$

$$A^g_\mu(x) = g^{-1}(x)(A_\mu(x) - i\partial_\mu)\,g(x) \quad , \quad g(x) \in U(n) \quad , \quad g(x) \xrightarrow[|x| \to \infty]{} 1\!\!1 \quad ;$$

$$A_\mu(x) = T^a A^a_\mu(x) \quad , \quad tr(T^aT^b) = n\delta^{ab} \quad , \quad T^aT^b = \delta^{ab} + (d^{abc} + if^{abc})T^c \quad ;$$

$$\mu,\nu = 0,1,\ldots,D-1 \quad ; \quad a,b,c = 0,1,\ldots,n^2-1 .$$

$$(3)$$

Here T^a denote hermitian U(n)-generators with T^0 belonging to the U(1)-subalgebra. On the right hand side (r.h.s.) of (1), called Wess-Zumino effective action [2], $\omega^{(D+1)}_{ChS}(A)$ denotes the density of the well known D+1=odd dimensional Chern-Simons secondary class (e.g. [4]). Recently, a new beautiful interpretation of (1) was suggested [3]. The gauge-noninvariance of $\det\left[-i\not{\nabla}_{L,R}(A)\right]$ (1) instead as an anomaly was considered in [3] as a representation of the (infinite parameter) gauge group $\{\forall g(x), \text{ satisfying (3)}\}$ with a nontrivial 1-cocycle (nonlocal functional of $A_\mu(x)$) given by the Wess-Zumino action.

2. Since $\gamma^{(D+1)} = \pm 1$ in odd D, chiral (L-,R-) fermions do not exist in odd D. It was first observed in [5-7] that the relevant counterpart of the chiral anomaly (1) in odd D is the parity-violating anomaly (PVA) in gauge theories with massless fermions $\Psi(x)$. Parity (space-reflection) in odd D is defined as:

$$\Psi^P(x) = -i\gamma_1 \Psi(x^P) \quad , \quad A^P_\mu(x) = (A_0, -A_1, A_2, \ldots, A_{D-1})(x^P) \quad ,$$

$$x^P \equiv (x^0, -x^1, x^2, \ldots, x^{D-1}) \tag{4}$$

In this section we shall analyze conditions for occuring or absence of PVA assuming boundary conditions (2).

PVA are in fact contained (when the fermion mass $m \to 0$) in the following specific (but not correct in general, see (i) below) definition [8] of the logarithm of the determinant of $\not{\nabla}(A) = \gamma_\mu(\partial_\mu + iA_\mu(x))$ (elliptic, self-adjoint) (after subtracting the trivial infinite constant $\ln\det[-(m+i\not{\partial})]$):

$$\ln\det\left[-(m+i\not{\nabla}(A))\right] = \frac{1}{2}\ln\det\left[m^2 + \not{\nabla}^2(A)\right] - i\frac{\pi}{2}\eta_{\not{\nabla}}[A] -$$

$$-i\frac{m}{2}\int_0^\infty d\tau \left(\frac{\pi}{\tau}\right)^{1/2} e^{-\tau m^2} \, \mathrm{Tr}\left[\widetilde{\mathrm{Erfc}}\left(\tau^{1/2}\not{\nabla}(A)\right) - \widetilde{\mathrm{Erfc}}\left(\tau^{1/2}\not{\partial}\right)\right] \; ; \tag{5}$$

$$\widetilde{\mathrm{Erfc}}(\alpha) = 2\pi^{-1/2}\,\mathrm{sign}(\alpha)\int_{|\alpha|}^\infty d\beta\, e^{-\beta^2} \quad , \quad \not{\partial}^2 = -\partial^2 \; ;$$

$$\ln\det\left[m^2 + \not{\nabla}^2(A)\right] = -\int_0^\infty d\tau\,\tau^{-1} e^{-\tau m^2}\,\mathrm{Tr}\left[e^{-\tau\not{\nabla}^2(A)} - e^{-\tau\not{\partial}^2}\right] \; . \tag{6}$$

$$\eta_{\not{\nabla}}[A] = \int_0^\infty d\tau\,(\pi\tau)^{-1/2}\,\mathrm{Tr}\left[\not{\nabla}(A)e^{-\tau\not{\nabla}^2(A)} - \not{\partial}\, e^{-\tau\not{\partial}^2}\right] \tag{7}$$

By means of the asymptotic ($\tau \to 0$) Seeley expansion [9]:

$$\exp\{-\tau\not{\nabla}^2(A)\}(x,x) = \sum_{j=0}^\infty \tau^{\frac{1}{2}(j-D)}\,\Phi^{(D)}_{\frac{1}{2}(j-D)}\left(\not{\nabla}^2(A);x\right) \tag{8}$$

one can check the absence of singularities at $\tau = 0$ (i.e. ultraviolet divergences) in the "proper time" integrals in (5) - (7) for odd D. Well-definiteness of the operator traces in (5) - (7) is guaranteed by the existence and completeness of the wave operators $U_\pm(\not{\nabla}^2(A); -\partial^2)$ (in

the sense of scattering theory [10]) for boundary conditions (2). $\eta_{\not{\nabla}}[A]$ denotes the well known η-invariant [11] of $\not{\nabla}(A)$ and under gauge-(3) and parity-(4) transformations:

$$\eta_{\not{\nabla}}[A^g] = \eta_{\not{\nabla}}[A] \quad , \quad \eta_{\not{\nabla}}[A^P] = -\eta_{\not{\nabla}}[A] \quad . \tag{9}$$

Therefore, appearence of $\eta_{\not{\nabla}}[A]$ on the r.h.s. of (5) gives rise to PVA when $m \to 0$.

Now, let us try to find a finite local counterterm (i.e. a local functional of $A_\mu(x)$) which, when added to the r.h.s. of (5), could elliminate PVA. In case of (2), starting from (7) and using (8) one arrives at the following important identity (for $\not{\nabla}(A)$ on compact manifolds, see [11,12]):

$$\delta/\delta A^a_\mu(x)\,\eta_{\not{\nabla}}[A] = (-1)^{\frac{1}{2}(D+1)}\, 2\,\delta/\delta A^a_\mu(x)\, W^{(D)}_{ChS}[A] + 2i\,\mathrm{tr}[T^a\gamma_\mu \mathcal{P}_{\not{\nabla}(A)}(0;x,x)] \quad ; \tag{10}$$

where $\mathcal{P}_{\not{\nabla}(A)}(\lambda;x,y)$ denotes the kernel of the density of the $\not{\nabla}(A)$-spectral projector. Using once again the existence and completeness of the wave operators $U_\pm(\not{\nabla}^2(A), -\partial^2)$ we find in the case of (2):

$$(0;x,x) = \begin{cases} \delta(0)\,\Pi_0^{\not{\nabla}(A)}(x,x) & \text{(if } \not{\nabla}(A) \text{ has zero modes, i.e. if } \lambda=0 \text{ is a discrete eigenvalue of } \not{\nabla}(A)) \\ 0 & \text{(if } \not{\nabla}(A) \text{ has only absolutely continious spectrum)} \end{cases} \tag{11}$$

where $\mathcal{P}_{\not{\nabla}(A)}(\lambda;x,y) = \delta(\lambda)\Pi_0^{\not{\nabla}(A)}(x,y) + \mathcal{P}'_{\not{\nabla}(A)}(\lambda;x,y)$ in case of zero modes with $\Pi_0^{\not{\nabla}(A)}$ being the projector on the $\lambda=0$ eigenspace and $\mathcal{P}'_{\not{\nabla}(A)}(0;x,y) = 0$. Combining (10),(11) one gets:

$$\eta_{\not{\nabla}}[A] = (-1)^{\frac{1}{2}(D+1)}\, 2W^{(D)}_{ChS}[A] + B[A] \quad , \tag{12}$$

where B[A] is a (piece-wise) constant functional of $A_\mu(x)$ taking values in the set of even integers and in general B[A] jumps by ± 2 at those $A_\mu(x)$ where $\not\nabla(A)$ acquires zero modes (in fact, B[A] may be associated to twice the index of an appropriate D+1=even-dimensional Dirac operator [11,12]).

Remembering the well known properties of $W^{(C)}_{ChS}[A]$ under (3),(4):

$$W^{(D)}_{ChS}[A^g] = W^{(D)}_{ChS}[A] + n_D[g] \quad , \quad W^{(D)}_{ChS}[A^P] = -W^{(D)}_{ChS}[A] \quad ,$$

$$n_D[g] = -(i/2\pi)^{1/2(D+1)} \left(\frac{D-1}{2}\right)!\,(D!)^{-1} \int d^Dx\, \varepsilon_{\mu_1 \ldots \mu_D} \mathrm{tr}\left[(g^{-1}\partial_{\mu_1} g)\ldots(g^{-1}\partial_{\mu_D} g)\right] , \tag{13}$$

and from (9) we have for B[A] (12):

$$B[A^g] = B[A] + (-1)^{1/2(D-1)} 2n_D[g] \quad , \quad B[A^P] = -B[A] \quad . \tag{14}$$

Note also the following property of the "winding number" $n_D[g]$ (13):

$$n_D[g] \begin{cases} \in \mathbb{Z} & \text{if } \pi_D(U(n)) = \mathbb{Z} \text{ , i.e. for odd } D < 2n, \\ = 0 & \text{if } \pi_D(U(n)) \neq \mathbb{Z} \text{ , i.e. for odd } D > 2n. \end{cases} \tag{15}$$

Now we can redefine (5) in a parity-preserving (when $m \to 0$) form by adding a finite local counterterm $\pm i\pi W^{(D)}_{ChS}[A]$ (cf. (12)) to the r.h.s. of (5) hereafter we assume that in general there are $N_f \geq 1$ fermion "flavors", i.e. $\not\nabla(A)_{ij} = \delta_{ij}\not\nabla(A)$, $i,j=1,\ldots,N_f$):

$$N_f \ln \widetilde{\det}[-(m+i\not\nabla(A))] = N_f \ln \det[-(m+i\not\nabla(A))]\,(\text{Eq.}(5)) + i\pi(-1)^{1/2(D+1)} N_f W^{(D)}_{ChS}[A] =$$

$$= 1/2 N_f \ln\det[m^2 + \not\nabla^2(A)] - iN_f \frac{\pi}{2} B(A) - \tag{16}$$

$$- i/2\, m N_f \int_0^\infty d\tau \left(\frac{\pi}{\tau}\right)^{1/2} e^{-\tau m^2}\, \mathrm{Tr}\left[\widetilde{\mathrm{Erfc}}(\tau^{1/2}\not\nabla(A)) - \widetilde{\mathrm{Erfc}}(\tau^{1/2}\not\partial)\right] .$$

Accounting for (14),(15) we arrive at the following conclusions from (16):

(i) If $2n < D$ or/and N_f=even (16) yields both gauge- and parity-invariant $\ln\widetilde{\det}[-i\not\nabla(A)]$, i.e. PVA are absent (for boundary conditions (2)). Therefore, definition (5) is not correct under the above conditions.

(ii) If $2n > D$ and N_f=odd simultaneously, the addition of $\pm i\pi W^{(D)}_{ChS}[A]$ to the r.h.s. of (5) breaks gauge invariance in (16). Therefore, the gauge invariant definition (5) should be retained and thus PVA are inavoidable in this case.

Under conditions (i) parity (3) may still be broken spontaneously as in the case of parity-invariant D=3 gauge nonlinear sigma models with fermions or D=3 supersymmetric nonlinear sigma models [13,14], where in some phases a nonzero fermion mass $m \sim \langle\bar\Psi\Psi\rangle$ is dynamically generated.

3. Now, let us assume that conditions (i) above are fulfilled but we shall abandon boundary conditions (2), i.e. we shall assume that:

$$F_{\mu\nu}(A) \xrightarrow[|x| \to \infty]{} F^\infty_{\mu\nu} \neq 0 \text{ (in certain or in all directions in } \mathbb{R}^D) \quad , \tag{17}$$

which means that $A_\mu(x)$ is an external background (not quantized) field. For (i) and (17) new types of PVA may arise (cf. [14] for D=3).

Here a remark is in order. In case of (17) the wave operators $U_\pm(\not\nabla^2(A);-\partial^2)$ do not already exist and, therefore, the operator traces in (5) - (7),(16) are not well defined. The standard trick is to consider first $\not\nabla(A)$ and (5) - (7) on a compact subset $K \subset \mathbb{R}^D$ (of finite volume V) and to impose appropriate boundary conditions for $A_\mu(x)$ on ∂K which should reproduce (17) when $V \to \infty$. In order to analyze the new PVA it is sufficient to consider the induced fermion current $J^a_\mu(x)$ (instead of $\ln\det[-(m+i\not\nabla(A))]$) which possesses a well defined $V \to \infty$ limit:

$$N_f^{-1} J_\mu^a(x) = N_f^{-1} \sum_{j=1}^{N_f} \langle \bar{\psi}_j(x) T^a (-i\gamma_\mu) \psi_j(x) \rangle = \lim_{V \to \infty} (-i)\, \delta/\delta A_\mu^a(x)\, \ln \widetilde{\det}[-(m + i\not{D}(A))]^{(V)} =$$

$$= \int_0^\infty d\tau\, e^{-\tau m^2}\, \mathrm{tr}\left\{ T^a \gamma_\mu \left[(\not{D}(A) + im) e^{-\tau \not{D}^2(A)} \right]_{(x,x)} \right\} -$$

$$- i\pi[1 + \mathrm{sign}(m)]\, \mathrm{tr}\left[T^a \gamma_\mu \mathcal{P}_{\not{D}(A)}(0;x,x) \right] = -i\pi[1 + \mathrm{sign}(m)]\, \mathrm{tr}\left[T^a \gamma_\mu \mathcal{P}_{\not{D}(A)}(0;x,x) \right] +$$

$$+ \int_{-\infty}^{\infty} d\lambda (\lambda + im)(\lambda^2 + m^2)^{-1}\, \mathrm{tr}\left[T^a \gamma_\mu \mathcal{P}_{\not{D}(A)}(\lambda;x,x) \right] \quad . \tag{18}$$

(18) directly follows from (16),(6) and the spectral decomposition of $\not{D}(A)$. Now taking the limit $m \to 0$ in (18) we get:

$$\lim_{m \to 0} N_f^{-1} J_\mu^a(x) = \int_{-\infty}^{\infty} d\lambda\, P(1/\lambda)\, \mathrm{tr}\left[T^a \gamma_\mu \mathcal{P}_{\not{D}(A)}(\lambda;x,x) \right] - \tag{19}$$
$$- i\pi\, \mathrm{tr}\left[T^a \gamma_\mu \mathcal{P}_{\not{D}(A)}(0;x,x) \right] \quad .$$

According to (4):

$$\mathcal{P}_{\not{D}(A^P)}(\lambda;x,y) = -\gamma_1 \mathcal{P}_{\not{D}(A)}(-\lambda;x^P,y^P)\gamma_1 \quad ,$$

hence the first term on the r.h.s. of (19) is of normal parity and the second one represents the general form of PVA in case of boundary conditions (17), i.e. PVA assuming (i) of Section 2. occur if and only if:

$$\mathcal{P}_{\not{D}(A)}(0;x,x) \neq 0 \quad (< \infty) \quad . \tag{20}$$

(Cf. (20) and (11): $\mathcal{P}_{\not{D}(A)}(0;x,x) = \infty$ in case of zero modes of $\not{D}(A)$ is just the usual infrared divergence of $\lim_{m \to 0} J_\mu^a(x)$.)

There are two particular cases when one can readily prove that (20) is satisfied:

(A) For static A_μ, i.e. x^o-independent, with zero electric field ($F_{ok}(A)=0$, $k=1,\ldots,D-1$; then the gauge $A_o=0$ may be imposed):

$$\mathcal{P}_{\not{D}_D}(0;x,x) = (2\pi)^{-1} \Pi_0^{\not{D}_{D-1}}(\underline{x},\underline{x}) \quad ; \tag{21}$$

$$\not{D}(A) \equiv \not{D}_D = \gamma_0 \partial_0 + \not{D}_{D-1} \quad , \quad \not{D}_{D-1} = \gamma_K(\partial_K + iA_K(\underline{x})) \quad , \quad \underline{x} \equiv (x^1,\ldots,x^{D-1}) \in \mathbb{R}^{D-1} ;$$

$$A_K(\underline{x}) = -i\hat{h}(\underline{x}_\infty)(\partial_K \hat{h})(\underline{x}_\infty) + O(|\underline{x}|^{-1-\varepsilon}) \qquad (|\underline{x}| \to \infty) \quad , \tag{2'}$$

$$\hat{h} : S_\infty^{D-2} \longrightarrow U(n) \quad , \quad \underline{x}_\infty \in S_\infty^{D-2} \quad .$$

The existence of zero modes of $\not{D}_{D-1}$ ($D-1$=even) with boundary conditions (2') is guaranteed by the index theorem (e.g. [4]). Substituting (21) into (19) and remembering that

$$\gamma_0(D=\mathrm{odd}) = i(-1)^{1/2(D+1)} \gamma^{(D)}(D-1=\mathrm{even}) \quad , \quad \gamma_k(D=\mathrm{odd}) = \gamma_k(D-1=\mathrm{even}) \quad ,$$

we get (for =o, i.e. the gauge-singlet current):

$$N_f^{-1} J_0^{a=0}(\underline{x}) = (-1)^{1/2(D+1)}\, 1/2\, \mathrm{tr}\left[\gamma^{(D)} \Pi_0^{\not{D}_{D-1}}(\underline{x},\underline{x}) \right] = (-1)^{1/2(D+1)}\, 1/2\, \mathrm{index}(\not{D}_{D-1};\underline{x}) \tag{22a}$$

(i.e. entirely parity-anomalous);

$$N_f^{-1} J_k^{a=0}(\underline{x}) = \pi^{-1/2} \int_0^\infty d\alpha\, \mathrm{tr} \left| \gamma_k \left(\not{D}_{D-1} e^{-\alpha^2 \not{D}_{D-1}^2} \right)_{(\underline{x},\underline{x})} \right. \quad \text{(i.e. parity-normal)}, \tag{22b}$$

where $\mathrm{index}(\not{D}_{D-1};\underline{x})$ indicates the index density of $\not{D}_{D-1}$. From (22a) a fractional induced charge in a static background $A_k(\underline{x})$ results (for D=3, see [7,15]):

$$Q_{ind} = \int d\underline{x}^{D-1} J_0^{a=0}(\underline{x}) = (-1)^{1/2(D+1)}\, 1/2\, N_f\, \mathrm{index}(\not{D}_{D-1}) = (-1)^{1/2(D+1)} N_f\, 1/2\, n_{D-2}[\hat{h}] \quad , \tag{23}$$

where $n_{D-2}[\hat{h}]$ is the "winding number" of $\hat{h}$ (2') and the last equality in (23) comes from the index theorem. Combining conditions (i) with (15) we deduce that in fact (23) yields half-integer Q_{ind} when $D=2n+1$ and N_f=odd, otherwise Q_{ind} is integer (or zero).

(B) Consider now in D=3 $F_{\mu\nu}(A) \to F_{\mu\nu}^\infty$ = constant uniform field-strength at $|x| \to \infty$ sufficiently fast. Noting that for $F_{\mu\nu}(A) = F_{\mu\nu}^\infty$:

$$\mathcal{P}_{\not\nabla_{D=3}}(0;x,x') = (16\pi^2)^{-1}\epsilon_{\mu\nu\lambda}(-i\gamma_\mu)F^{\infty}_{\nu\lambda}$$

(cf. e.g. [16]) the induced current (19) reads:

$$\lim_{m\to 0} N_f^{-1} J^a_\mu(x) = n(8\pi)^{-1}\epsilon_{\kappa\lambda\nu}F^{\infty,b}_{\lambda\nu} W^{ab}_{\mu\kappa}(x) +$$

$$W^{ab}_{\mu\kappa}(x) = -(2n)^{-1}\, tr\left[T^a\gamma_\mu\left(\int d^3y\, U_\pm(x,y)\right)T^b\gamma_\kappa\left(\int d^3y\, U^*_\pm(y,x)\right)\right] =$$

$$= \delta_{\mu\kappa}\delta^{ab} + \qquad (24)$$

Here $U_\pm = U(\not\nabla^2(A);\not\nabla^2(A^{as}))$ and $F_{\mu\nu}(A^{as}) = F^{\infty}_{\mu\nu}$. Let us point out that the whole $\overline{\text{PVA}}$ in (24) comes entirely from the nonzero asymptotic value of $F_{\mu\nu}(A)$ which agrees with the statement (i) of Section 2.

4. Let us return to boundary conditions (2) and consider the opposite limit $|m|\to\infty$ (fermion decoupling) in (18), i.e. assuming (i) of Section 2. (this limit is equivalent to the low-energy or slowly varying A_μ-field limits):

$$\lim_{|m|\to\infty} N_f^{-1} J^a_\mu(x) = \lim_{|m|\to\infty} m^{-1}\int_0^\infty d\alpha\, e^{-\alpha}\, tr\left\{T^a\gamma_\mu\left[\left(m^{-1}\not\nabla(A)+i\right)e^{-(\alpha/m^2)\not\nabla^2(A)}\right](x,x)\right\} =$$

$$= sign(m)\, i\pi^{1/2}\, tr\left[T^a\gamma_\mu\,\Phi^{(D)}_{-1/2}(\not\nabla^2(A);x)\right] =$$

$$= sign(m)(-1)^{\frac{1}{2}(D-1)}\,\pi\,\frac{\delta}{\delta A^a_\mu(x)} W^{(D)}_{ChS}[A] \qquad (25)$$

where (8) was used and $\Phi^{(D)}_{-1/2}(\not\nabla^2(A);x)$ is straightforwardly computed. From (25) the exact parity-anomalous heavy fermion effective action results:

$$\lim_{|m|\to\infty} N_f \ln \widetilde{\det}[-(m+i\not\nabla(A))] = i\pi\, sign(m)(-1)^{\frac{1}{2}(D-1)} N_f W^{(D)}_{ChS}[A] \quad . \qquad (26)$$

(26) was previously found as a low-energy effective action in the context of the $1/N_f$ expansion of D=3 gauge nonlinear sigma models with fermions [13] where the r.h.s. of (26) (for D=3):

$$\pm i(16\pi)^{-1}N_f\,\epsilon_{\mu\nu\lambda}\int d^3x\, tr\left[A_\mu F_{\nu\lambda}(A) - i\,\tfrac{2}{3}A_\mu A_\nu A_\lambda\right]$$

is nothing but a dynamically generated (in the phases with dynamical parity breakdown due to $m\sim\langle\bar\Psi\Psi\rangle\neq 0$) topological gauge invariant mass term [17] for A_μ.

Finally, let us compare (26) (where conditions (i) of Section 2. were assumed) with the corresponding results for the m limit of (5), i.e. assuming (ii) of Section 2.:

$$\lim_{|m|\to\infty} \ln\det[-(m+i\not\nabla(A))] = i\pi[1+sign(m)](-1)^{\frac{1}{2}(D-1)} W^{(D)}_{ChS}[A] =$$

$$= \begin{cases} 2i\pi(-1)^{\frac{1}{2}(D-1)} W^{(D)}_{ChS}[A] & (m\to\infty)\ , \\ 0 & (m\to-\infty)\ . \end{cases} \qquad (27)$$

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